MTP-P&VE-T-64-1
January 22, 1964

N64-17670*

CODE-1

TMX-51481

32p.

GEORGE C. MARSHALL SPACE FLIGHT CENTER

HUNTSVILLE, ALABAMA

1613702

MONTE CARLO APPLICATION FOR DEVELOPING A DESIGN RELIABILITY GOAL COMPATIBLE WITH SMALL SAMPLE REQUIREMENTS

By

Ray Heathcock
and
Dale L. Burrows

22 JAN. 1964 32p refs

(NASA TMX-51481;

MTP-P&VE-T-64-1)

OTS; $3.60ph, $1.16mf

OTS PRICE

XEROX $ 3.60 ph

MICROFILM $ 1.16 mf.

Marshall Space Flight Center, Huntsville, Ala.

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ABSTRACT

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The reliability of many items in space vehicles (from piece parts to large structural elements) can be appropriately considered to be the result of probability interaction between the distributions of strength (failure) and stress (load). This investigation applies Monte Carlo simulation to the construction of empirical sampling distributions of reliability for various sample sizes taken from the strength and stress distributions, which are assumed Gaussian. Emphasis has been placed on very small sample sizes and very high values of reliability since these are situations commonly imposed on space vehicle development programs because of the costs involved.

The empirical sampling distribution of reliability estimates appear to be very sensitive to the ratio of the standard deviations of the stress and strength distributions; therefore, specific sampling distributions are constructed for selected values of this ratio.

It is concluded from this investigation that the constructed empirical sampling distributions, can be utilized to aid the designer in establishing a design reliability goal, place a confidence coefficient on reliability estimates, and to determine if sample data, taken from the stress and strength distributions, demonstrates a specified reliability at a specified confidence level.

Author

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TECHNICAL AND SCIENTIFIC STAFF
PROPULSION AND VEHICLE ENGINEERING LABORATORY

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DEFINITION OF SYMBOLS

Symbols	Definition
X_1	A variate from the strength distribution.
X_2	A variate from the stress distribution.
M_{x_1}	Population mean of strength distribution.
M_{x_1}	Population mean of stress distribution.
$\overline{X}_1$	Sample mean of strength distribution.
$\overline{X}_2$	Sample mean of stress distribution.
σx_1	Population standard deviation of strength distribution.
σx_2	Population standard deviation of stress distribution.
S_{x_1}	Sample standard deviation of strength distribution.
S_{x_2}	Sample standard deviation of stress distribution.
N_{x_1}	Sample of size N from the strength distribution.
N_{x_2}	Sample of size N from the stress distribution.
Z	A normal deviate
K_c	A normal deviate representing a specified confidence. K_c is obtained from the sampling distribution $\dfrac{\overline{X}_1 - \overline{X}_2}{\sqrt{S_{x_1}{}^2 + S_{x_2}{}^2}}$
P_r	Probability
R	Reliability

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SUMMARY

This report describes the application of Monte Carlo simulation to constructing the empirical sampling distribution of reliability estimates obtained by sampling from the classical stress and strength (load and failure) distributions, which are assumed Gaussian. Typical stress/strength distributions, representing specific values of reliability, were stored in a computer. From these distributions various sample sizes were taken and the resulting estimate of reliability computed. Iteration of this process resulted in the construction of empirical sampling distributions for specific values of reliability and specific sample sizes. Emphasis was placed on very high values of reliability (.99 to .99999999) and on very small sample sizes (2 to 8) because a high reliability requirement coupled with a limited number of test articles is commonly imposed on space vehicle development programs.

Since the sampling distribution of reliability estimates was found to be very sensitive to the ratio of the standard deviations of the stress and strength distributions, sampling distributions was constructed for specific values of this ratio. Since the true ratio of standard deviations will seldom be known, the variation in the sampling distribution, due to this ratio, introduces a certain weakness into the application of the analysis. This weakness is discussed fully in the body of this report.

It is concluded from this investigation that the sampling distributions, constructed by Monte Carlo simulation, may be utilized to aid the designer in establishing a design reliability goal, place a confidence coefficient on reliability estimates, and to determine if sample stress and strength data demonstrates a specified reliability at a specified confidence level.

The primary purpose of this report is to present a method and examples of the use of the method. The ogives given in this report may be used in actual application, however, not indescriminately, since the ogives contain small inaccuracies due to the curve fitting procedure.

SECTION I. INTRODUCTION

A. Background

In large space vehicle development, and in many other fields as well, a designer is often asked to design his equipment knowing that later a very small sample of items will be tested to determine if a specified minimum reliability is demonstrated at some fairly high confidence level. The designer is often quite perplexed as to the methods and reliability goals he should utilize in formulating his design. His own training and experience make him favor the safety factor approach, and yet statisticians and reliability engineers advocate safety margins, stress/strength relationships and other statistical approaches. The publications of Robert Lusser advocate the safety margin approach, (Ref 1) while ARINC Research Corporation under contract to NASA (Ref 2) has advocated the use of stress/strength relationships as a reliability prediction technique. There are many other papers and publications which propose the use of statistical variations in stress and strength in design and subsequent analysis of the design. The application of a single distribution (strength) and a reliability boundary (upper limit of stress) is discussed in references 1 and 3.

B. Scope

This investigation attempts to shed some additional light on problems concerning the use of stress/strength statistics in design, reliability demonstration, and confidence limits. It is primarily concerned with the solution of statistical sampling problems for which no theoretical solution was readily available and with the application of the data provided by these solutions. In this analysis, very small sample sizes in the range of 2 to 8 and high reliability values from .99 to .99999999 have been purposely used because space vehicle development programs are restricted to small samples and require high reliability values.

The authors wish to acknowledge the very capable assistance of Messrs. Robert Crafts, Joe Medlock and Matt Blue of the Computation Laboratory, MSFC, for programming the Monte Carlo simulation scheme, and of Mr. E. L. Bombara, Engine Projects Office, MSFC, whose technical advice and suggestions were very helpful. This paper was presented at the U. S. Army Ninth Conference on Design of Experiment at the Redstone Arsenal October 23, 1963.

SECTION II. DESCRIPTION

A. Analysis

The reliability of many items in large space vehicles from piece parts to large structural elements, can be appropriately considered to be a function of a stress distribution and a strength distribution. A stress distribution is defined as a distribution of stresses to which the population of items will be subjected in actual use, and a strength distribution is defined as a distribution of stresses which will cause failure of the items. This analysis applies to the general problem of pitting a distribution of "what an item will do" against a distribution of "what it is required to do" in any performance parameter. The analysis is not restricted to the more common application of structural stress/strength analysis.

An example of the relationship between the stress and strength distributions of a typical situation is shown in Figure 1. The distributions are assumed to be normal, an assumption which is retained throughout this paper.

A randomly chosen value of stress (X_2) subtracted from a randomly chosen value of strength (X_1) gives a variate from the strength minus stress distribution. By repeating this process many times, a distribution of strength minus stress may be formed. It will appear as shown in Figure 2. The mean of this distribution is the mean of the strength distribution minus the mean of the stress distribution. The standard deviation of this distribution is the square root of the sum of the variances of the strength and stress distributions. At some point on this distribution, stress equals strength and a zero point appears on the $X_1 - X_2$ axis.

Since any negative value of strength minus stress represents a failure, the area below the zero point represents unreliability, and the area above the zero point represents reliability. Mathematically this may be stated as follows (Ref 4):

$$R = P_r\,(X_1 \geqq X_2)$$

$$R = P_r\,(X_1 - X_2 - M_{x_1} + M_{x_2} \geqq - M_{x_1} + M_{x_2})$$

$$R = P_r\,\left(\frac{X_1 - X_2 - M_{x_1} + M_{x_2}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}} \geq \frac{M_{x_2} - M_{x_1}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}\right)$$

$$R = P_r\,\left(Z \geqq \frac{M_{x_2} - M_{x_1}}{\sigma_{x_1}^2 + \sigma_{x_2}^2}\right) \quad \text{since Z is a normal deviate}$$

In actual practice the parameters of the stress and strength distribution will rarely be known and must be estimated from sample data. Small sample estimates of the parameters may be used to estimate reliability as follows:

$R_{est} = P_r\,\left(Z \geq \frac{\overline{X}_2 - \overline{X}_1}{\sqrt{S_{x_1}^2 + S_{x_2}^2}}\right)$ which can be obtained from the normal table of areas.

Since an estimate of the reliability of a particular situation can now be made, the next logical step is to describe the variations expected in this estimate due to sampling. In order to make use of the stress/strength statistical relationship, the sampling distribution of reliability estimates, based on this relationship, must be developed. Since there was no known theoretical solution to the description of this variation, a computer program was developed using Monte Carlo simulation to derive the empirical sampling distribution of the quantity $\frac{\overline{X}_2 - \overline{X}_1}{\sqrt{S_{x_1}^2 + S_{x_2}^2}}$.

Very briefly, this simulation technique consists of the following steps:

(1) Store hypothetical stress and strength distributions in the computer.

(2) Generate a pseudo random number.

(3) Use this random number to get a random value of strength (X_1) repeating this process N times.

(4) Compute a sample mean and standard deviation for strength.

(

(5) Repeat the process for stress.

(6) Compute $K = \dfrac{\bar{X}_2 - \bar{X}_1}{\sqrt{S_{x_1}{}^2 + S_{x_2}{}^2}}$

(7) Form a histogram of values of K, which represents the sampling distribution. As many values of K as desired may be obtained from the program, dependent on the accuracy desired. 1000 values were used to obtain the information for this paper. The sampling distribution may be put in a cumulative form, termed an ogive, in order to be able to read K values corresponding to selected values of probability.

B. Results

The factors which influence the sampling distribution of K will now be discussed. First, sample size, of course, will influence it. An empirical sampling distribution must be generated for each sample size that will be used in actual practice. To give an idea of how the sampling distribution varies, as sample size varies, some ogives for various sample sizes have been developed.

The ogives in figure 3, from left to right, represent values of reliability from .99 to .99999999. In other words, the first curve to the left represents the variation inherent in estimating a true .99 reliability using a specified sample size. This figure represents the case where $N_{x_1} = 8$ and $N_{x_2} = 8$ i.e., sample sizes of 8 from each of the strength and stress distributions. The standard deviations of stress and strength are equal in this case. The effect of varying these standard deviations will be discussed later. Other ogives for various sample sizes and equal sigmas appear in figures 4 through 8.

As may be observed from figures 3 through 8, the ogives vary quite radically with sample size, especially in the very small sample sizes used here. In order to use this type empirical data, ogives for the specific sample sizes used in a particular application must be developed.

The second important factor, which causes the sampling distribution to vary, is the ratio of the standard deviations of stress and strength. This ratio is defined as the smaller standard deviation divided by the larger standard deviation, regardless if the standard deviation is from stress or strength. Early in this program Monte Carlo results indicated that the sigma ratio had an important effect on the variation of the sampling distribution, and later, the theoretical application of reference 5 served to verify this conclusion. The results of the theoretical application showed that for equal sample sizes the degrees of freedom of the non-central t distribution is a function of sample size and the ratio of standard deviations of the stress and strength distributions. Figure 9 shows the variations in the sampling distributions for various ratios of the standard deviations with a fixed sample size of $N_{x_1} = N_{x_2} = 5$. Here only the upper portion of the distributions are shown since most applications will be concerned with high confidence and because the variation is most pronounced in this area. These curves were obtained by holding all variables in the computer program constant except the sigma ratios. The sample sizes were equal for stress and strength. As the figure indicates by convergence of the curves, there is little variation in the sampling distribution of "K" attributable to sigma ratios below the 80th percentile point of "K". Above this point there is considerable variation; the higher ratios result in sampling distributions which are skewed to a greater degree. Although ratios up to 1/100 were run on the Monte Carlo program, the practical range is probably between 1 and 1/10 for most applications. Even at the 1/10 ratio, however, there is a large variation; therefore, separate curves for the specific ratio must be utilized in actual application. A weakness, however, in this procedure is that in practice the actual ratio will seldom be known and therefore must be estimated from sample sata. An F test for the ratio of two variances could be used to establish if there is any significant difference between the sigmas and possibly, confidence limits for the ratio of two variances could be utilized for estimating the limits of the ratios. Further work is desirable in developing an approach to establishing a ratio from sample data which could be used for entering the appropriate set of curves.

Another important factor to consider is the case of unequal sigmas (stress and strength) and unequal sample sizes. The most skewed condition (long tail to the right) in the sampling distribution of "K" will result when the smaller sample is taken from the distribution which has the larger standard deviation. For instance, assume that the standard deviation of strength is twice that of stress and a sample of 8 is taken from the stress distribution and 5 from the strength distribution, i.e., the smaller sample is taken from the distribution with the larger sigma.

What would happen if a reverse procedure were used and the large sample taken from the distribution having the larger sigma?

The ogive for this case is shown in figure 10. The broken line represents the ogive for the large sample matched with large sigma, and the solid line represents a rerun of the same case with the small sample matched with the large sigma. As can be seen, there is a significant variation in the two conditions. It is concluded, therefore, that this is another condition which must be included in the Monte Carlo output in order to utilize the results efficiently and accurately. This presents no problem, however, if the sigma ratio is known since Monte Carlo runs for the required conditions can be made. It is being presented merely to illustrate that it does have an influence.

The discussion, thus far, has dealt only with the mechanics of obtaining the sampling distribution of "K" and its variation as related to specific factors causing it. It is appropriate to discuss the application of such data.

C. Application

The criteria for determining whether sample data demonstrates a given reliability at a specified confidence will be developed in terms of "K" which is a normal deviate corresponding to a specified area under the normal curve.

The basis for developing the demonstration criteria is shown in Figure 11 which depicts the distribution of $X_1 - X_2$ and $X_1 - X_2 - K_C \sqrt{S_{x_1}^2 + S_{x_2}^2}$. The Z shown in the figure is a normal deviate, the area above which (obtained from normal tables of areas) represents the reliability which it is required to demonstrate. K_C represents a normal deviate greater than Z which must be found and when applied will assure a demonstrated reliability with confidence C.

If a K_C that satisfies the following inequality
$P_r\left(X_1 - X_2 - K_C \sqrt{S_{x_1}^2 + S_{x_2}^2} \geqq M_{x_1} - M_{x_2} - Z\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}\right) = C$
(Ref 3) can be found and applied as a criteria, a decision can be made as to whether or not the sample data demonstrates a specified reliability (Z). This inequality may be reduced to

$$P_r\left(X_1 - X_2 - K_C \sqrt{S_{x_1}^2 + S_{x_2}^2} \geqq 0\right) \text{ since } M_{x_1} - M_{x_2} - Z\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} = 0$$

in this situation, because the Monte Carlo program was run on this basis. Manipulation of this inequality, as follows, mathematically, gives the criteria:

$$P_r\left(X_1 - X_2 \leq K_c \sqrt{S_{x_1}^2 + S_{x_2}^2}\right) = C$$

or

$$P_r\left(\frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_{x_1}^2 + S_{x_2}^2}} \leq K_c\right) = C$$

Now K_c can be found by referring to the Monte Carlo developed ogives for the conditions of the problem (specified Z, N_{x_1} N_{x_2}, σ_{x_1} σ_{x_2}). Once K_c is found, the criteria for demonstration is as follows:

If the sample quantity $\dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_{x_1}^2 + S_{x_2}^2}} \geq K_c$, reliability Z is demonstrated with the desired confidence. As an example, suppose samples of $N_{x_1} = 8$, $N_{x_2} = 8$ gives $\bar{X}_1 = 80,000$, $\bar{X}_2 = 60,000$, $S_{x_1} = 3000$, $S_{x_2} = 3000$, and the problem was to determine if .9999 reliability (Z = 3.71) was demonstrated at the 90% confidence level. The 90% point on the ogives for .9999, $N_{x_1} = N_{x_2} = 8$, $\sigma_{x_1} = \sigma_{x_2}$ is found to be 4.95 which is K_c. $\dfrac{\bar{X}_1 - \bar{X}_2}{S_{x_1}^2 + S_{x_2}^2} = 4.7$ therefore since $4.7 < 4.95$ reliability .9999 is not demonstrated at the 90% confidence level. The accuracy of the answer obtained is dependent on how close the true sigma ratio is to unity, since it was assumed that $\sigma_{x_1} = \sigma_{x_2}$ in this example.

Another application of the Monte Carlo results is the establishment of a lower confidence limit on a reliability estimate. The appropriate set of ogives is entered with the reliability estimate (reliability estimate expressed in terms of a K value) and a confidence is read for each reliability value represented by a curve. If sample data ($N_{x_1} = N_{x_2} = 8$, $\sigma_{x_1} = \sigma_{x_2}$) gave a reliability estimate expressed in terms of K = 3.5 for a given sigma ratio and a given sample size, proceed as follows to arrive at a lower confidence limit:

(1) Refer to the series of ogives (figure 12) representing the sigma ratio and sample sizes applicable to the problem.

(2) On the horizontal scale locate a K value of 3.5 which was the sample estimate and draw a vertical line through the point.

(3) Where the vertical line intersects a curve, read a confidence for the reliability represented by that curve. For the example, figure 12 shows 95% confidence in .99 reliability, 70% confidence in .999 reliability, etc. By generating a sufficient number of curves, a confidence coefficient for the reliability estimate in question, or at least close enough to it for practical application, can be obtained.

If in the initial design of an item, a designer knew exactly the stress distribution and knew exactly what strength distribution he could get, designing would be a simple problem and there would be no need for demonstration. However, this is not the case, and the designer has to make estimates of the distribution.

Assuming that the designer knew exactly the strength and stress distributions, and was to design, knowing that a small sample of his items was to be tested later for demonstration purposes, it would be to his benefit to overdesign so that he would have a good chance, say 90%, of having the small sample demonstrate the specified reliability. In order to arrive at how much he should overdesign, he can consult the Monte Carlo developed ogives and find a K_c such that if the sample K is greater than K_c, the sample data has demonstrated the required reliability at the desired confidence. Reference to figure 13 will aid the reader in following this approach. If the designer wants a 90% chance of having a sample demonstrate the required reliability, he must design to a reliability represented by an ogive 90% of which is above the K_c point. This logic seems to be non sensical since if a designer knows the distributions he could just design to the reliability he desires and there would be no point in a demonstration program; however, this logic can be applied to the situation where the designer does not know the distributions, but has some knowledge of them from experience or design calculation. Assuming a designer wants to design so that he has a 90% chance of demonstrating a specific reliability with a specified sample size, it can be concluded from the previous discussion that he must design to a reliability above that which gives him a 90% chance of demonstrating it with the specified sample size. How much above is a matter of engineering judgement, and depends upon how well the designer thinks he can estimate the distribution. This conclusion vividly points out the fact that a design goal (inherent design reliability) must be higher than the specific reliability which is to be demonstrated at a high degree of confidence. There are, of course, other factors which influence the establishment of reliability goals. Weight, cost and performance all should influence design decisions and must be properly considered as tradeoff factors against the statistically developed goal.

Since much of the foregoing discussion of reliability has been in terms of the normal deviate Z, figure 14 has been provided to enable the reader to determine Z directly from a numerical value of unreliability (1-R).

SECTION III. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

This investigation has revealed that the statistical information afforded by very small samples from a stress/strength situation (even as low as 2) can be useful to the designer and a reliability engineer. It has also revealed that the sampling distribution of reliability estimates made by taking sample data from the stress/strength distributions is very sensitive to sample size and the ratio of the standard deviation of the stress and strength distributions.

It is concluded that the empirical sampling distribution of reliability estimates can be utilized by the designer or reliability engineer as follows:

1. Reliability Demonstration - Given sample data from stress and strength distributions, a determination can be made as to whether the sample data demonstrates a specified reliability at a chosen confidence level.

2. Establishing Confidence Coefficients - Given sample data from a stress/strength situation, confidence coefficients for various values of reliability can be estimated, limited only by the number of curves that have been generated by Monte Carlo.

3. Recommend Design Goal - One can establish and recommend to the designer a design goal such that if he designs it, a pre-chosen sample will demonstrate a specified reliability at a chosen confidence level, a specified percent of the time.

B. Recommendations for Future Investigation

In view of the number of promising applications discovered during this investigation and the weaknesses and limitations that are inherent in the nature and extent of this analysis, it is believed that the following areas are worthy of further investigation:

1. Since the ratio of standard deviations of the stress and strength distributions has a large affect on the variance of the reliability sampling distribution (Distribution of "K") and since in practice this ratio will seldom be known but must be estimated from sample data, a method should be derived for establishing a ratio which could be used for entering the appropriate set of empirical curves.

2. Extension of the analysis to various combinations of different types of distributions, other than normal, should be investigated by the Monte Carlo process since in these areas not even approximations to theoretical solutions are available.

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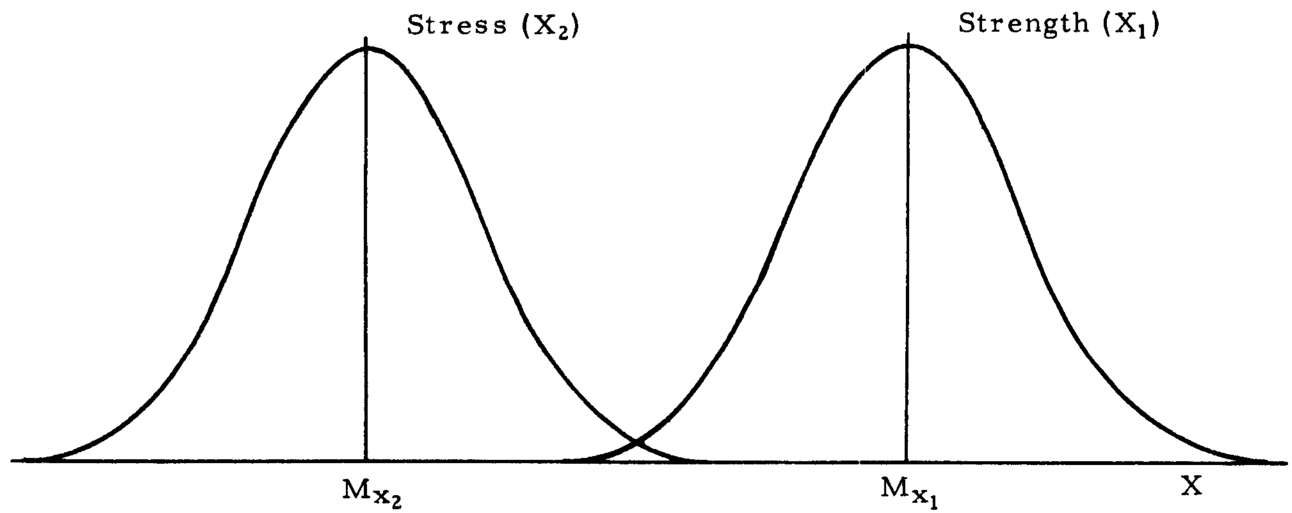
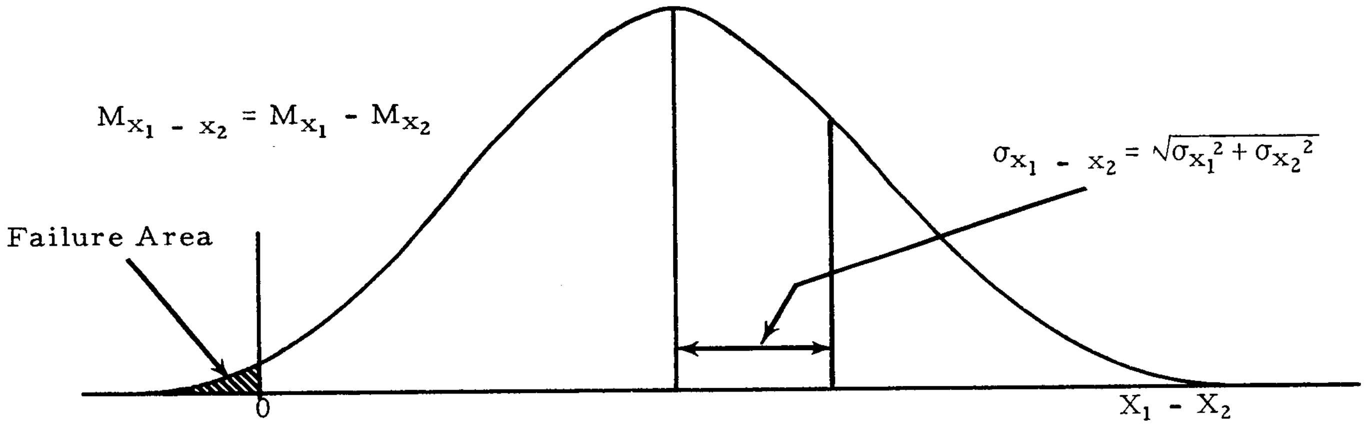


FIGURE 1. RELATIONSHIP OF STRENGTH AND STRESS DISTRIBUTIONS (NORMAL)



Reliability = $P_r \; (X_1 > X_2)$

$R = P_r \; (X_1 - X_2 - M_{x_1} + M_{x_2} > -M_{x_1} + M_{x_2})$

$R = P_r \left(\frac{X_1 - X_2 - M_{x_1} + M_{x_2}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}} > \frac{M_{x_2} - M_{x_1}}{\sqrt{\sigma_{x_2}^2 + \sigma_{x_2}^2}} \right)$

$R = P_r \left(Z > \frac{M_{x_2} - M_{x_1}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}} \right)$ Where: Z = Normal deviate

$R_{est} = P_r \left(Z > \frac{\overline{X}_2 - \overline{X}_1}{\sqrt{S_{x_1}^2 + S_{x_2}^2}} \right)$ Normal table of areas

FIGURE 2. RELIABILITY COMPUTATION - STRENGTH/STRESS DISTRIBUTIONS

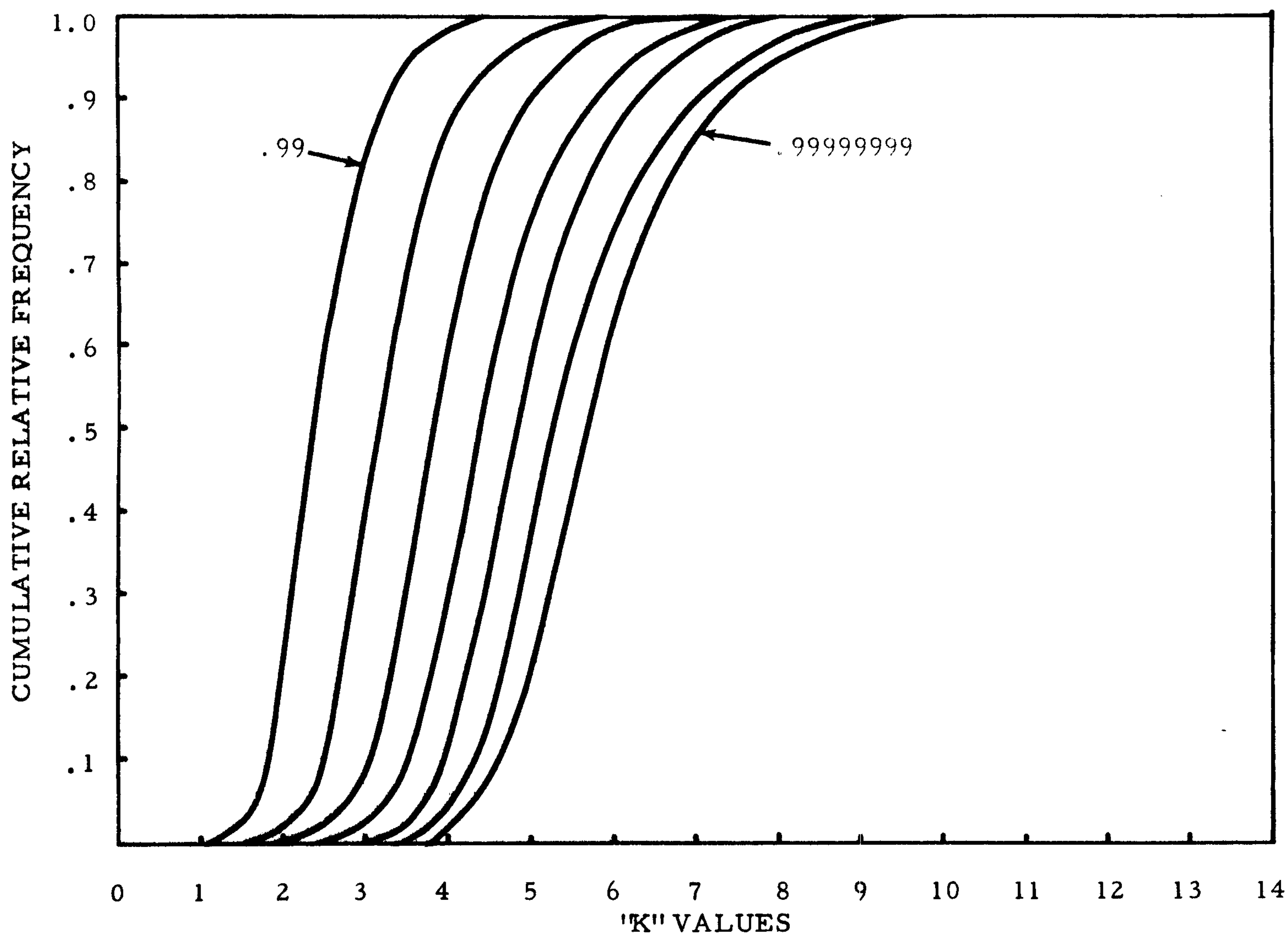


FIGURE 3. SAMPLING DIST. OF "K", $N_{x_1} = N_{x_2} = 8$, $\sigma_{x_1} = \sigma_{x_2}$

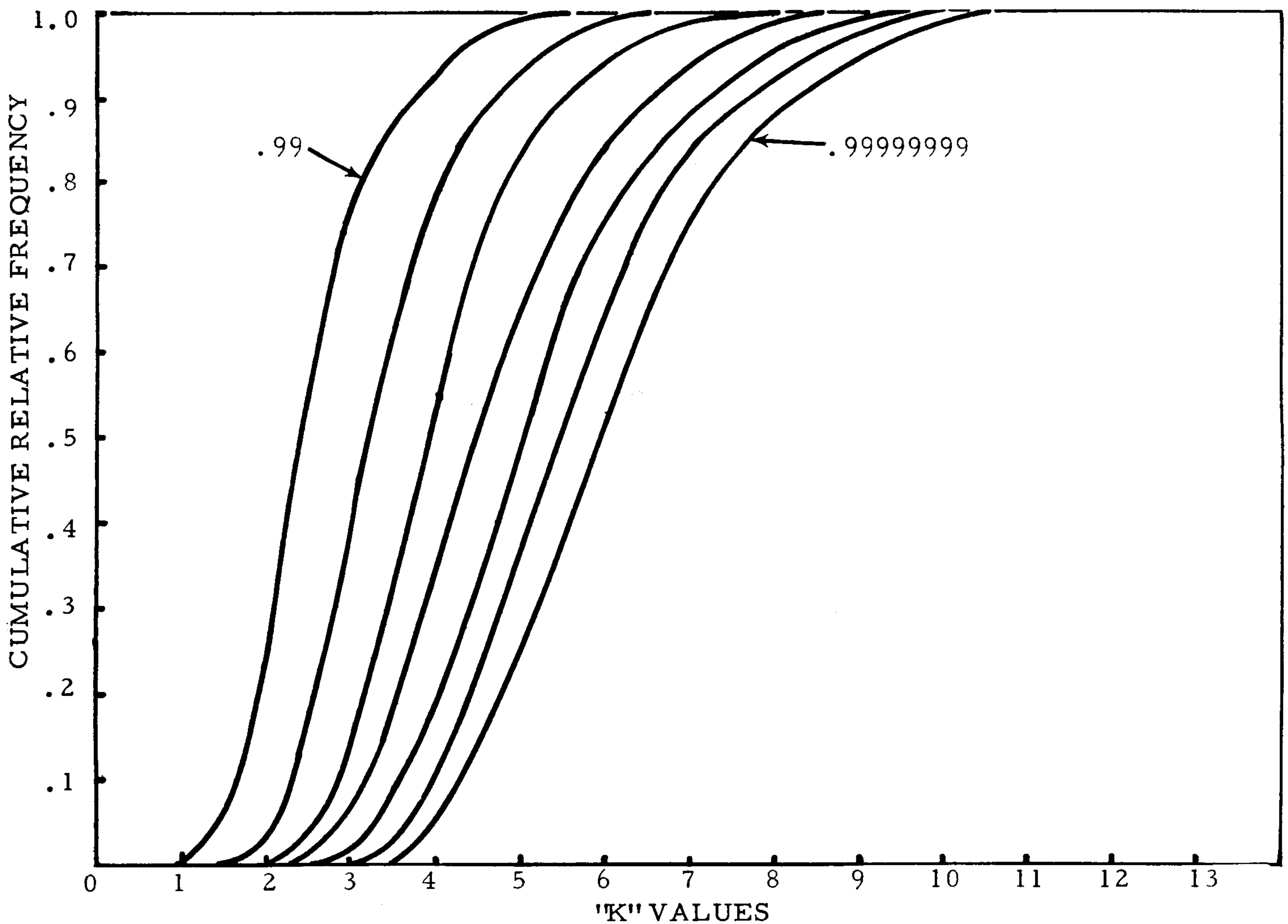


FIGURE 4. - SAMPLING DIST. OF 'K', $N_{x_1} = N_{x_2} = 5$, $\sigma_{x_1} = \sigma_{x_2}$

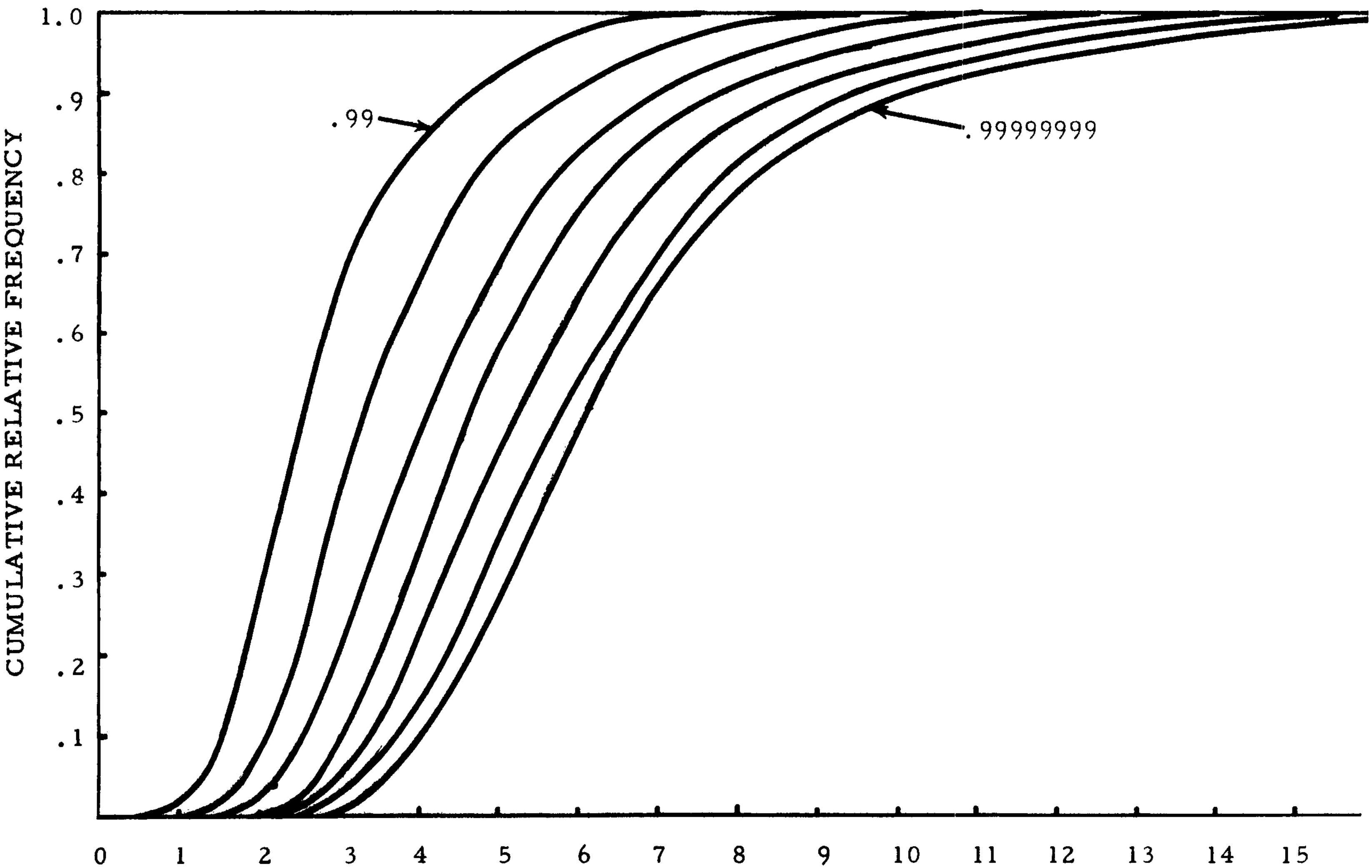


FIGURE 5. SAMPLING DIST. OF 'K", $N_{x_1} = N_{x_2} = 3$, $\sigma_{x_1} = \sigma_{x_2}$

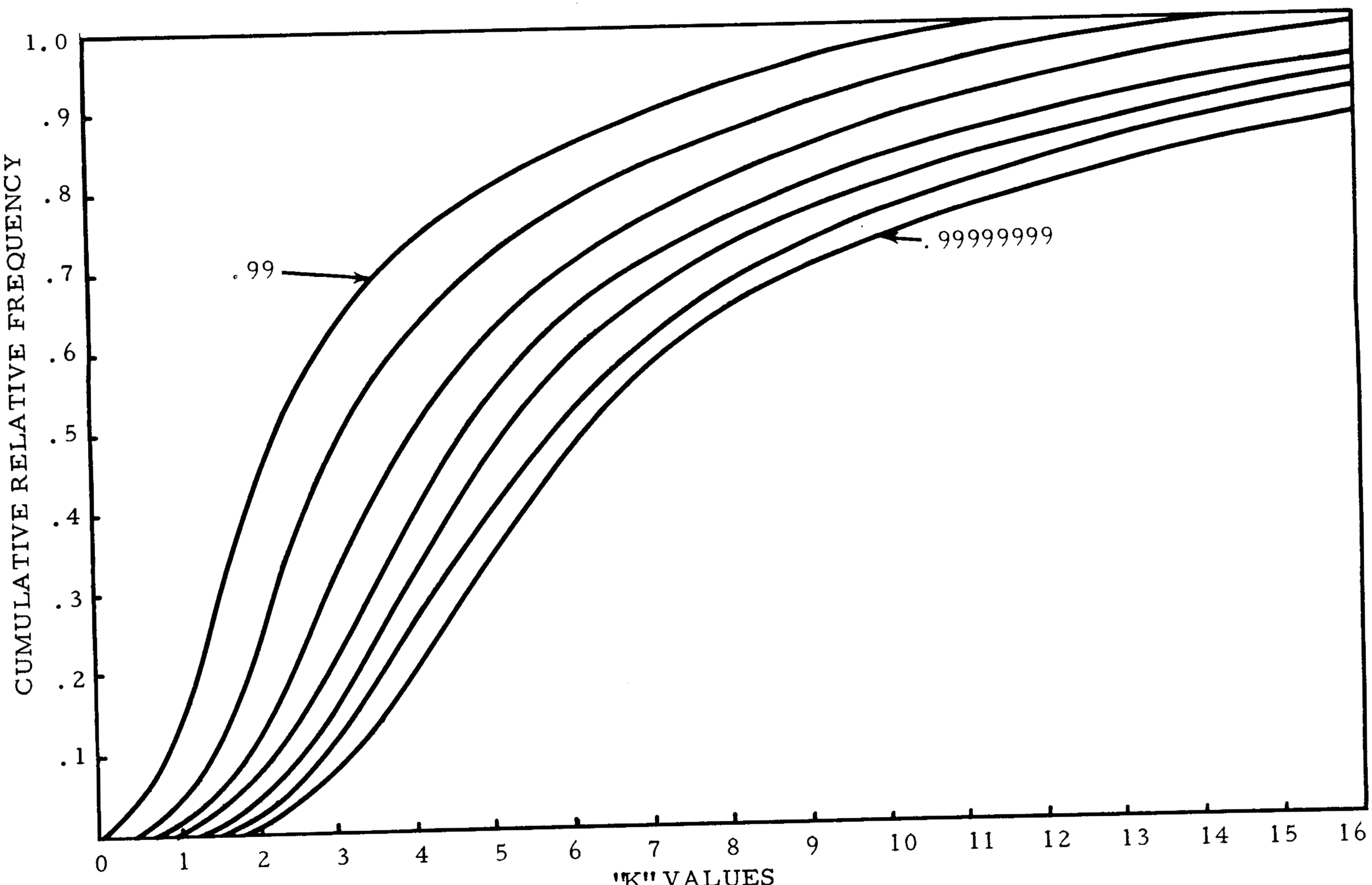


FIGURE 6. SAMPLING DIST. OF "K", $N_{x_1} = N_{x_2} = 2$, $\sigma_{x_1} = \sigma_{x_2}$

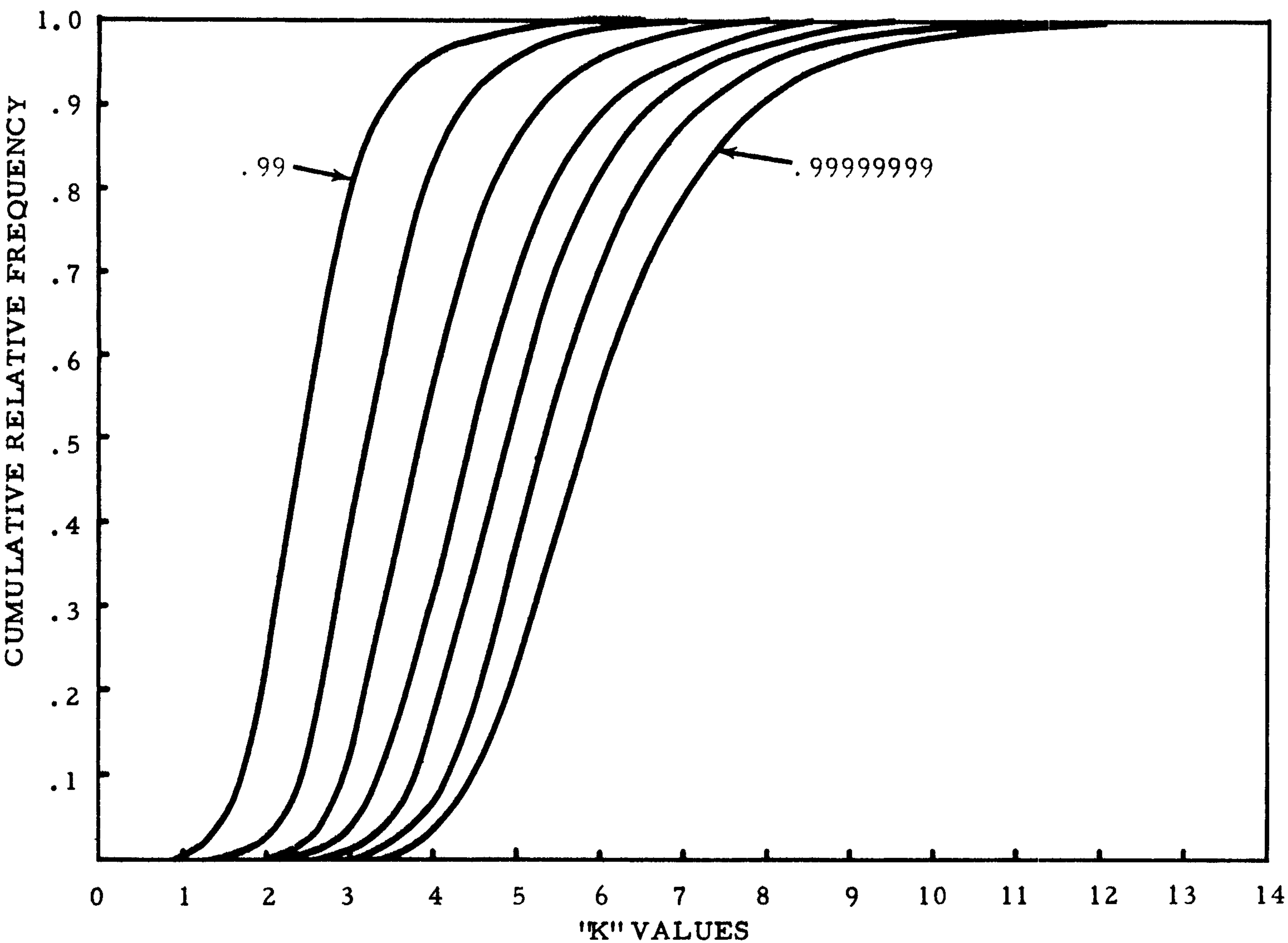


FIGURE 7. SAMPLING DIST. OF "K", $N_{x_1} = 8$, $N_{x_2} = 5$, $\sigma_{x_1} = \sigma_{x_2}$

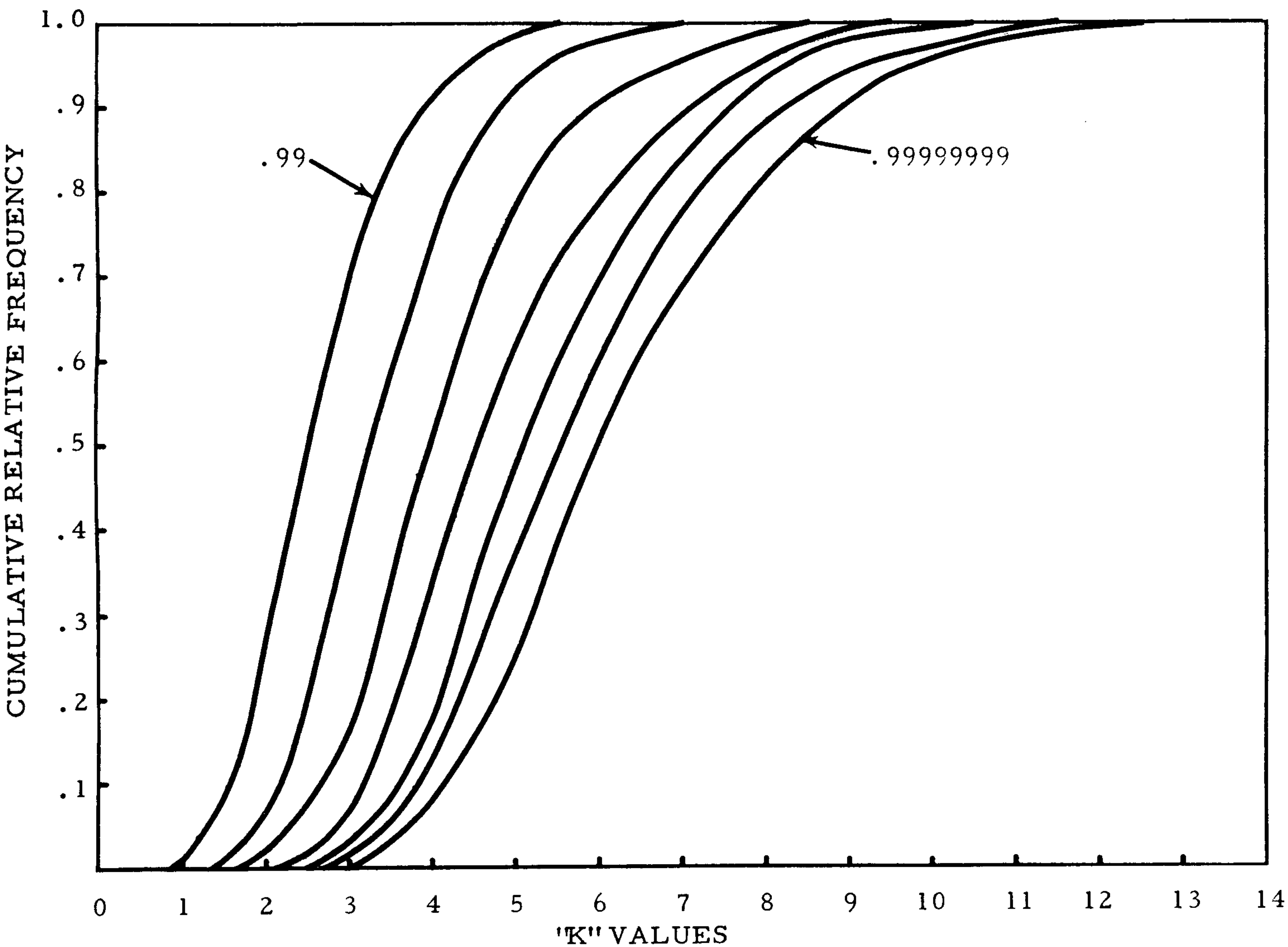


FIGURE 8. SAMPLING DIST. OF 'K", $N_{x_1} = .5$, $N_{x_2} = 3$, $\sigma_{x_1} = \sigma_{x_2}$

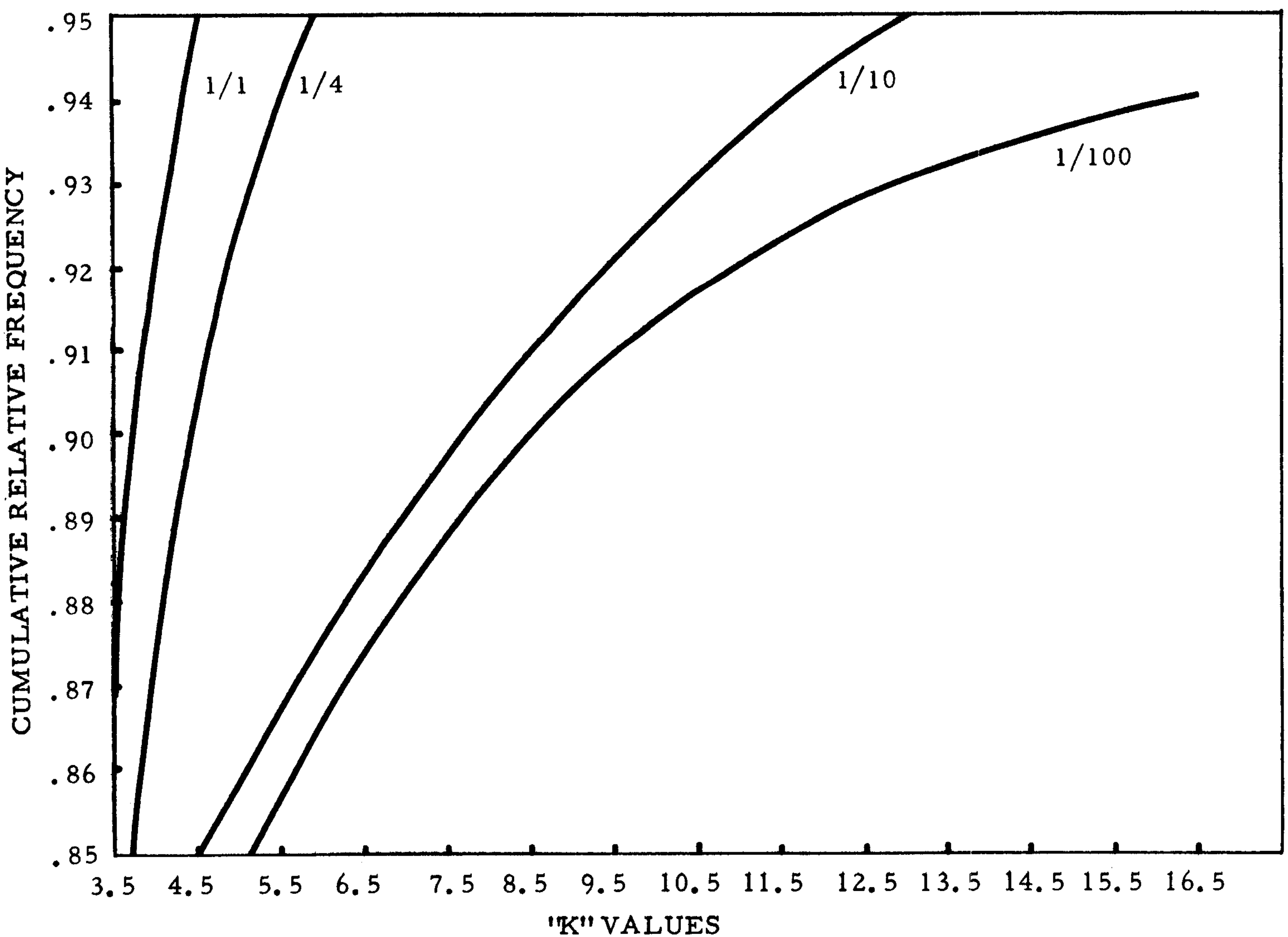


FIGURE 9. VARIATIONS IN DIST. OF "K" DUE TO SIGMA RATIO

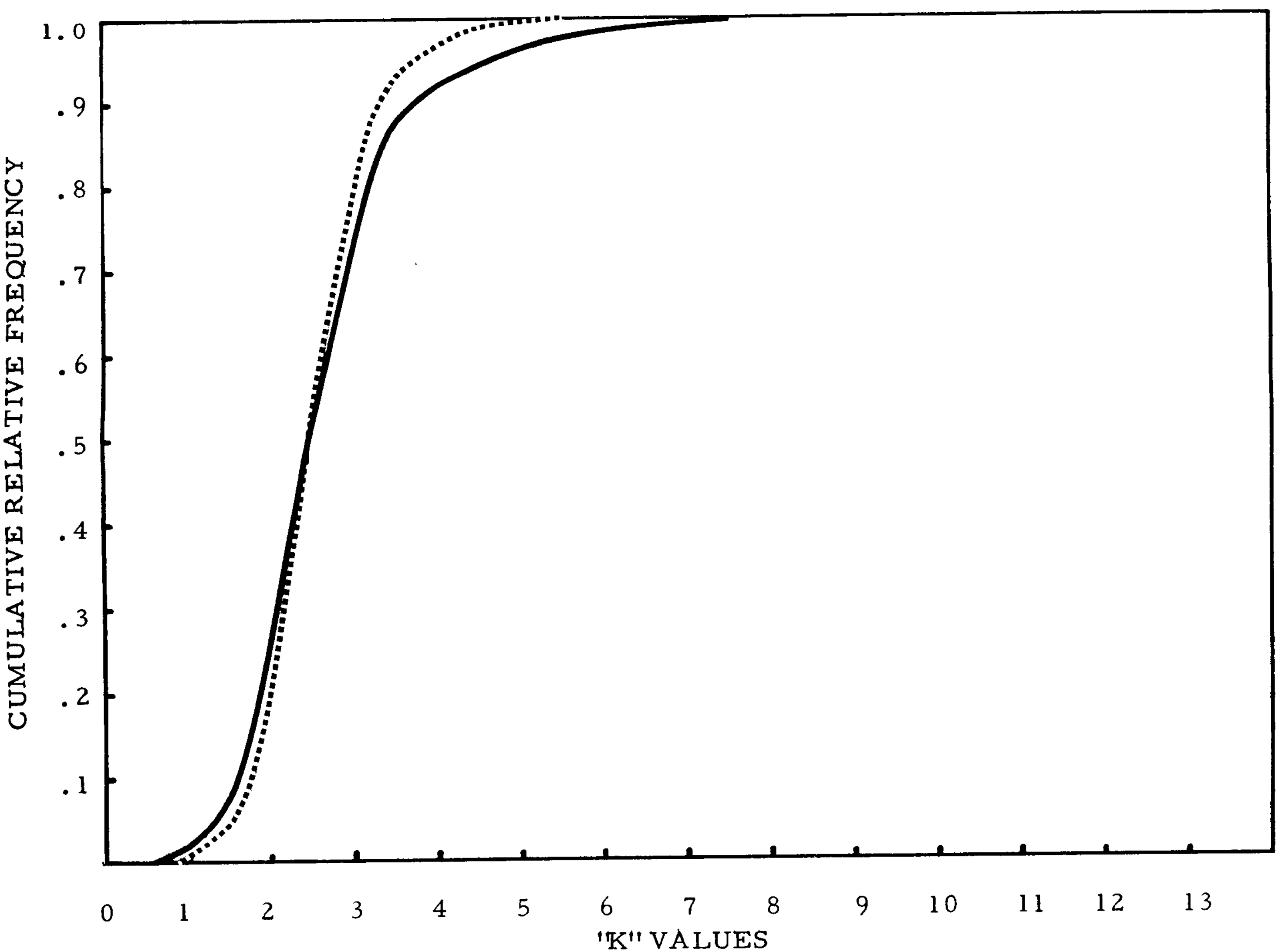
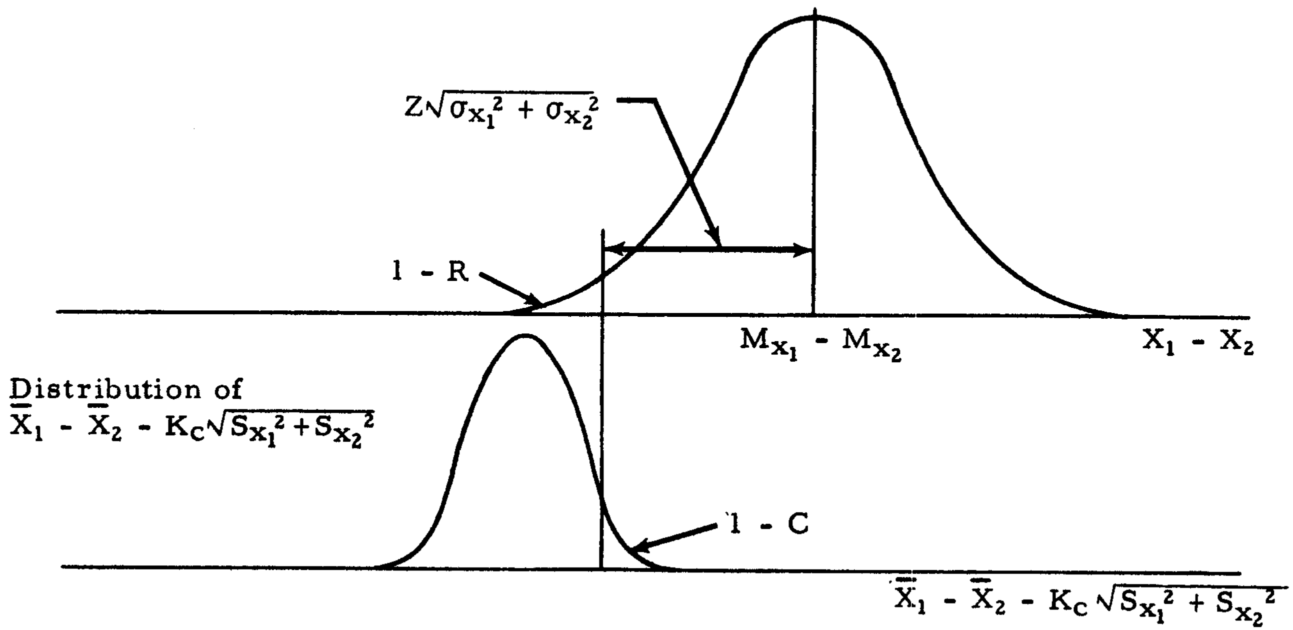


FIGURE 10. SAMPLING DIST. OF "K", $N_{x_1} \neq N_{x_2}$, $\sigma_{x_1} \neq \sigma_{x_2}$



$P(\overline{X}_1 - \overline{X}_2 - K_C \sqrt{S_{x_1}^2 + S_{x_2}^2} \leqq M_{x_1} - M_{x_2} - Z\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}) = C$

$P(\overline{X}_1 - \overline{X}_2 \leqq K_C \sqrt{S_{x_1}^2 + S_{x_2}^2}) = C$ Since $M_{x_1} - M_{x_2} - Z\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2} = 0$

$P\left(\frac{\overline{X}_1 - \overline{X}_2}{\sqrt{S_{x_1}^2 + S_{x_2}^2}} \leqq K_C\right) = C$ K_C is the 90% point on the distribution of $\frac{\overline{X}_1 - \overline{X}_2}{\sqrt{S_{x_1}^2 + S_{x_2}^2}}$

FIGURE 11. DISTRIBUTION OF $X_1 - X_2$ and $\overline{X}_1 - \overline{X}_2 - K_C \sqrt{S_{x_1}^2 + S_{x_2}^2}$

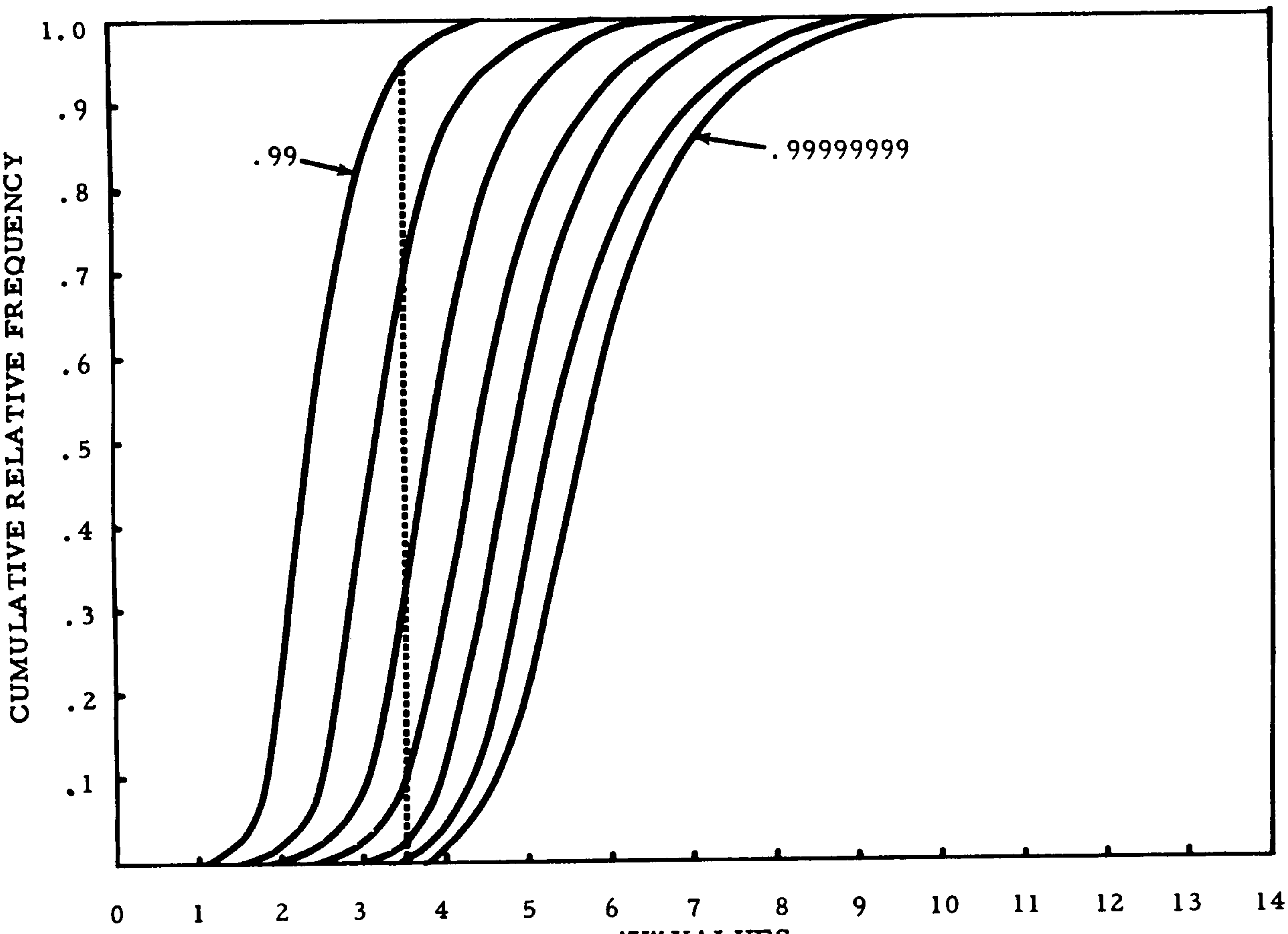


FIGURE 12. SAMPLING DIST. OF "K", $N_{x_1} = N_{x_2} = 8$, $\sigma_{x_1} = \sigma_{x_2}$

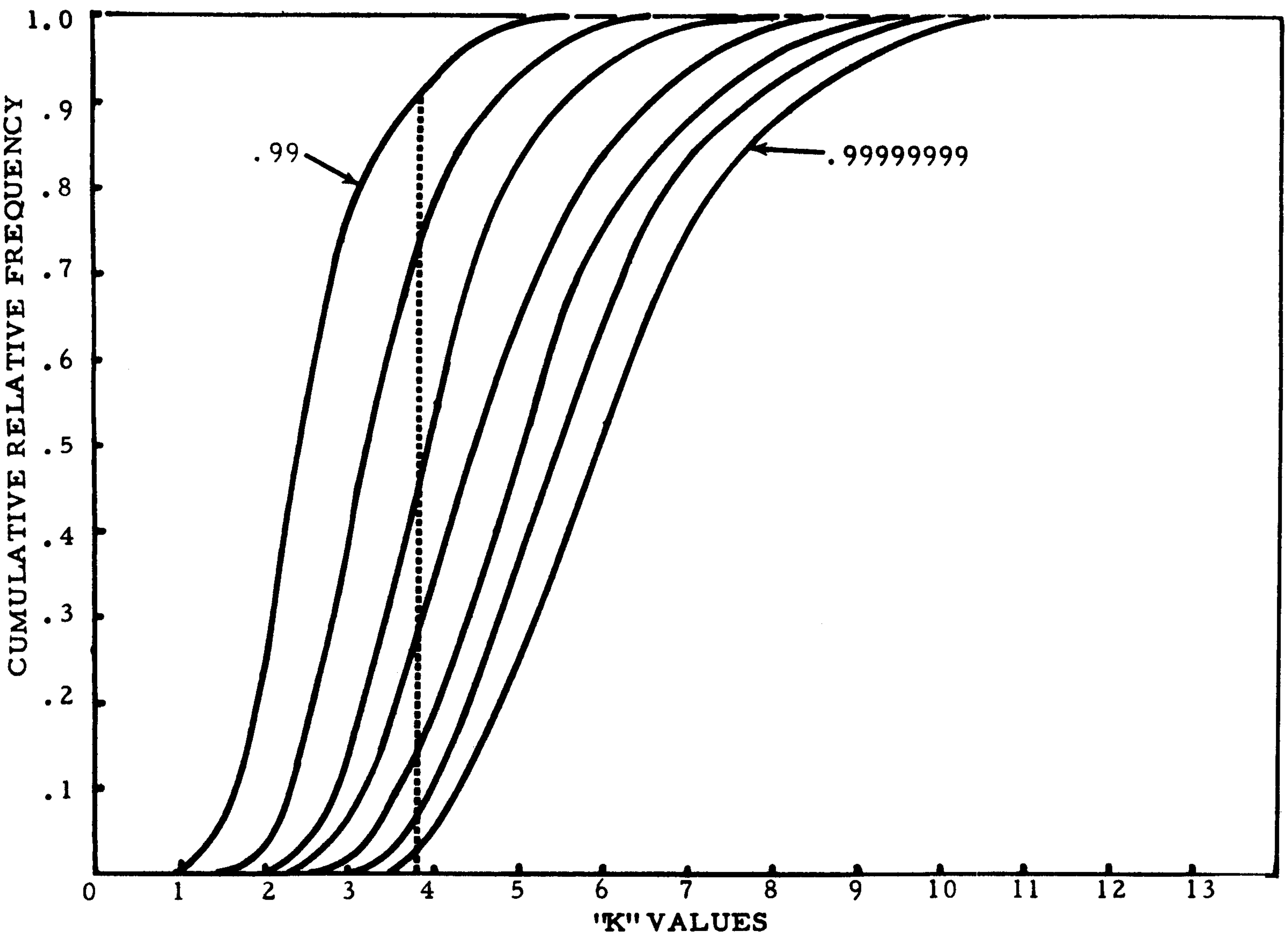


FIGURE 13.- SAMPLING DIST. OF "K", $N_{x_1} = N_{x_2} = 5$, $\sigma_{x_1} = \sigma_{x_2}$

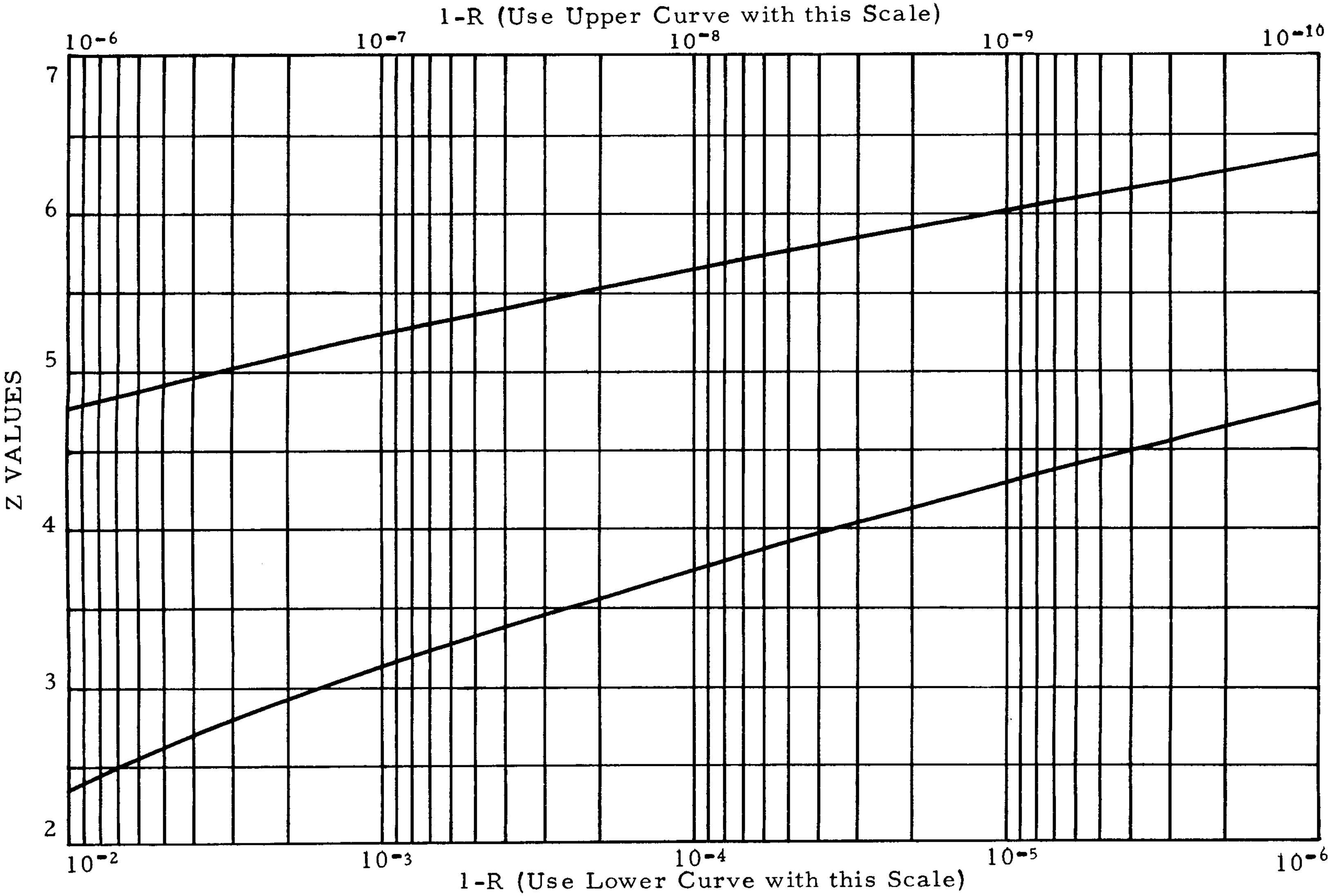


FIGURE 14. Graph of Z vs 1-R (Unreliability)

January 22, 1964 APPROVAL MTP-P&VE-T-64-1

MONTE CARLO APPLICATION FOR DEVELOPING A DESIGN RELIABILITY GOAL COMPATIBLE WITH SMALL SAMPLE REQUIREMENTS

By

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